

13.5: 4, 10, 14, 18, 30

$$\begin{aligned} 4. \text{ (a) } \operatorname{curl} \mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & \cos xz & -\sin xy \end{vmatrix} \\ &= (-x \cos xy + x \sin xz) \mathbf{i} - (-y \cos xy - 0) \mathbf{j} + (-z \sin xz - 0) \mathbf{k} \\ &= x(\sin xz - \cos xy) \mathbf{i} + y \cos xy \mathbf{j} - z \sin xz \mathbf{k} \\ \text{(b) } \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(\cos xz) + \frac{\partial}{\partial z}(-\sin xy) = 0 + 0 + 0 = 0 \end{aligned}$$

10. (a)  $\operatorname{curl} f = \nabla \times f$  is meaningless because  $f$  is a scalar field.

(b)  $\operatorname{grad} f$  is a vector field.

(c)  $\operatorname{div} \mathbf{F}$  is a scalar field.

(d)  $\operatorname{curl}(\operatorname{grad} f)$  is a vector field.

(e)  $\operatorname{grad} \mathbf{F}$  is meaningless because  $\mathbf{F}$  is not a scalar field.

(f)  $\operatorname{grad}(\operatorname{div} \mathbf{F})$  is a vector field.

(g)  $\operatorname{div}(\operatorname{grad} f)$  is a scalar field.

(h)  $\operatorname{grad}(\operatorname{div} f)$  is meaningless because  $f$  is a scalar field.

(i)  $\operatorname{curl}(\operatorname{curl} \mathbf{F})$  is a vector field.

(j)  $\operatorname{div}(\operatorname{div} \mathbf{F})$  is meaningless because  $\operatorname{div} \mathbf{F}$  is a scalar field.

(k)  $(\operatorname{grad} f) \times (\operatorname{div} \mathbf{F})$  is meaningless because  $\operatorname{div} \mathbf{F}$  is a scalar field.

(l)  $\operatorname{div}(\operatorname{curl}(\operatorname{grad} f))$  is a scalar field.

$$14. \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^z & 1 & xe^z \end{vmatrix} = (0 - 0) \mathbf{i} - (e^z - e^z) \mathbf{j} + (0 - 0) \mathbf{k} = \mathbf{0} \text{ and } \mathbf{F} \text{ is defined on all of } \mathbb{R}^3 \text{ with}$$

component functions that have continuous partial derivatives, so  $\mathbf{F}$  is conservative. Thus there exists a function  $f$  such that  $\nabla f = \mathbf{F}$ . Then  $f_x(x, y, z) = e^z$  implies  $f(x, y, z) = xe^z + g(y, z) \Rightarrow f_y(x, y, z) = g_y(y, z)$ . But  $f_y(x, y, z) = 1$ , so  $g(y, z) = y + h(z)$  and  $f(x, y, z) = xe^z + y + h(z)$ . Thus  $f_z(x, y, z) = xe^z + h'(z)$  but  $f_z(x, y, z) = xe^z$ , so  $h(z) = K$ , a constant. Hence a potential function for  $\mathbf{F}$  is  $f(x, y, z) = xe^z + y + K$ .

18. No. Assume there is such a  $\mathbf{G}$ . Then  $\operatorname{div}(\operatorname{curl} \mathbf{G}) = xz \neq 0$  which contradicts Theorem 11.

30.  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \Rightarrow r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$ , so

$$\mathbf{F} = \frac{\mathbf{r}}{r^p} = \frac{x}{(x^2 + y^2 + z^2)^{p/2}} \mathbf{i} + \frac{y}{(x^2 + y^2 + z^2)^{p/2}} \mathbf{j} + \frac{z}{(x^2 + y^2 + z^2)^{p/2}} \mathbf{k}$$

Then  $\frac{\partial}{\partial x} \frac{x}{(x^2 + y^2 + z^2)^{p/2}} = \frac{(x^2 + y^2 + z^2) - px^2}{(x^2 + y^2 + z^2)^{1+p/2}} = \frac{r^2 - px^2}{r^{p+2}}$ . Similarly,

$\frac{\partial}{\partial y} \frac{y}{(x^2 + y^2 + z^2)^{p/2}} = \frac{r^2 - py^2}{r^{p+2}}$  and  $\frac{\partial}{\partial z} \frac{z}{(x^2 + y^2 + z^2)^{p/2}} = \frac{r^2 - pz^2}{r^{p+2}}$ . Thus

$$\begin{aligned} \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{r^2 - px^2}{r^{p+2}} + \frac{r^2 - py^2}{r^{p+2}} + \frac{r^2 - pz^2}{r^{p+2}} = \frac{3r^2 - px^2 - py^2 - pz^2}{r^{p+2}} \\ &= \frac{3r^2 - p(x^2 + y^2 + z^2)}{r^{p+2}} = \frac{3r^2 - pr^2}{r^{p+2}} = \frac{3-p}{r^p} \end{aligned}$$

Consequently, if  $p = 3$  we have  $\operatorname{div} \mathbf{F} = 0$ .